

University of Washington

Department of Mathematics

Midterm Examination

MATH 394: Probability I

Summer 2026 — Instructor: Arman Jahangiri

Date: July 22, 2026

Answering time: 60 minutes

Total points: 60

Name: _____

Student ID: _____

Signature: _____

Instructions

- This examination contains **five problems**. Verify that your copy is complete.
- Show all work and justify your answers. Unsupported answers may receive little or no credit.
- Clearly define any events or random variables that you introduce.
- Do not remove pages from the examination booklet.

Permitted materials

Examinations are closed-book and closed-notes. Students may use:

- one handwritten 8.5×11 inch reference sheet (front and back), and
- a non-programmable non-graphing calculator capable of standard exponential and logarithmic computations (e.g. TI-30X IIS).

No other electronic devices, calculators, communication devices, or external resources may be used during examinations.

Problem	Score
1	/10
2	/10
3	/10
4	/15
5	/15
Total	/60

By signing above, you affirm that the work is your own and that you followed the examination rules.

Problem 1.**5 points**

A student organization has 7 mathematics graduate students, 5 statistics graduate students, and 6 undergraduate students. A committee of 5 students is selected uniformly at random from all committees of 5.

Find the probability that the committee contains exactly 2 mathematics graduate students, exactly 1 statistics graduate student, and exactly 2 undergraduate students.

Problem 2.**10 points**

A spam filter is designed by looking at commonly occurring phrases in spam. Suppose that 80% of email is spam. In 10% of the spam emails, the phrase “free money” is used, whereas this phrase is only used in 1% of non-spam emails. A new email has just arrived, which does mention “free money.” What is the probability that it is spam?

Problem 3.**10 points**

Let A and B be events.

- (a) Let S be a sample space and $A_1, \dots, A_n \in \mathcal{F}$ be events. Explain what it means for A_i 's to form a partition of S , **(3 points)**
 - (b) Suppose $P(A) = 0.6$ and $P(B) = 0.5$. Can A and B be disjoint? **(3 points)**
 - (c) Suppose that A is independent of itself. Use the definition of independence to conclude that $P(A) = 0$ or $P(A) = 1$. **(4 points)**
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Problem 4.**15 points**

Suppose that X has probability density function

$$f_X(x) = \begin{cases} cx(2-x), & 0 < x < 2, \\ 0, & \text{otherwise,} \end{cases}$$

where c is a constant.

- (a) Find c . **(3 points)**
 - (b) Find the cumulative distribution function $F_X(x)$ for all real x . **(5 points)**
 - (c) Find $P(X > 3/2)$. **(2 points)**
 - (d) Find $\mathbb{E}[X]$ and $\text{Var}(X)$. **(5 points)**
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Problem 5.**20 Points**

A company has found that 40% of customers who visit its website make a purchase. Suppose that 200 customers visit the website independently on a particular day.

Let X denote the number of customers who make a purchase.

1. Determine whether a **Poisson approximation** or a **normal approximation** is more appropriate by verifying the required conditions. Then clearly state the approximating distribution; that is, provide λ if a Poisson approximation is appropriate, or provide μ and σ^2 if a normal approximation is appropriate. **(5 points)**
2. Use the approximation from part (a) to estimate each of the following:
 - (a) $\mathbb{P}(70 < X < 90)$ **(5 points)**
 - (b) $\mathbb{P}(X > 90)$ **(5 points)**
 - (c) $\mathbb{P}(X = 80)$ **(5 points)**