University of Washington
Department of Mathematics

Midterm Examination (Practice)

MATH 394: Probability I

Summer 2026 — Instructor: Arman Jahangiri

Date:	July 22, 2026
Answering time:	60 minutes
Total points:	60

Name: _____

Student ID: _____

UW NetID: _____

Signature: _____

Instructions

- This examination contains **five problems**. Verify that your copy is complete.
- Show all work and justify your answers. Unsupported answers may receive little or no credit.
- Clearly define any events or random variables that you introduce.
- For any perceived ambiguity, state a reasonable interpretation and proceed.
- Do not remove pages from the examination booklet.

Permitted materials

Examinations are closed-book and closed-notes. Students may use:

- one handwritten 8.5 × 11 inch reference sheet (front and back), and
- a non-programmable non-graphing calculator capable of standard exponential and logarithmic computations (e.g. TI-30X IIS).

No other electronic devices, calculators, communication devices, or external resources may be used during examinations.

Problem	Score
1	/10
2	/10
3	/10
4	/15
5	/15
Total	**/60**

By signing above, you affirm that the work is your own and that you followed the examination rules.

Problem 1. **10 points**

A university student ID consists of three uppercase English letters followed by four digits. Letters and digits may repeat. The first letter cannot be `X`, and at least one of the four digits must be even.

How many such student IDs are possible? Justify your counting.

Problem 2. **10 points**

A warehouse receives items from three factories. Factory A supplies 40% of the items and has a defect rate of 2%. Factory B supplies 35% of the items and has a defect rate of 5%. Factory C supplies 25% of the items and has a defect rate of 8%.

An item is selected uniformly at random from the warehouse and is found to be defective.

(a) Find the probability that the item came from Factory C. **(6 points)**

(b) Find the probability that the item did not come from Factory A. **(4 points)**

Problem 3. **10 points**

Let A, B, and C be events.

(a) Prove that

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C).$$

(6 points)

(b) Deduce that

$$P(A \cup B \cup C) \leq P(A) + P(B) + P(C).$$

(2 points)

(c) Give a simple necessary and sufficient condition for equality in part (b). **(2 points)**

Problem 4. **15 points**

The cumulative distribution function of a continuous random variable X is

$$F_X(x) = \begin{cases} 0, & x < 0, \\ \dfrac{x^2}{4}, & 0 \le x < 1, \\ \dfrac{2x-1}{4}, & 1 \le x < 2, \\ 1 - \dfrac{(3-x)^2}{4}, & 2 \le x < 3, \\ 1, & x \ge 3. \end{cases}$$

(a) Find the probability density function $f_X(x)$. **(4 points)**

(b) Prove that $f_X(x)$ found in part (a) is a PDF. **(3 points)**

(c) Find $P\left(\frac{1}{2} < X \le \frac{5}{2}\right)$. **(3 points)**

(d) Find $\mathbb{E}[X]$. **(5 points)**

Problem 5. **15 points**

A city operates a large bike-share system. Assume that the outcomes of different rides are independent.

(a) On a typical day, 8000 rides are completed. Each ride results in a serious mechanical failure with probability 0.0005.

Let X be the number of serious mechanical failures during the day.

Determine whether a Poisson approximation or a Normal approximation to the distribution of X is more appropriate. Verify the relevant conditions and state the approximating distribution. **(3 points)**

(b) Use the approximation from part (a) to estimate the probability that at least 3 serious mechanical failures occur during the day. **(3 points)**

(c) At one busy station, 300 bikes are rented during the afternoon. Each bike is returned late with probability 0.35.

Let Y be the number of bikes returned late.

Determine whether a Poisson approximation or a Normal approximation to the distribution of Y is more appropriate. Verify the relevant conditions and state the approximating distribution. **(3 points)**

(d) Using the approximation from part (d), estimate

$$P(95 \leq Y \leq 115).$$

(3 points)
