

Probability Distribution Reference Sheet

MATH 394: Probability I (Summer 2026)

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Discrete Distributions

Name	Parameters	Probability Mass Function	Support	Mean	Variance	MGF
Bernoulli	$0 \leq p \leq 1$	$P(X = k) = p^k(1 - p)^{1-k}$	$x = 0, 1$	p	$p(1 - p)$	$1 - p + pe^t$
Binomial	$n \geq 1, 0 \leq p \leq 1$	$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$	$k = 0, \dots, n$	np	$np(1 - p)$	$(1 - p + pe^t)^n$
Poisson	$\lambda > 0$	$P(X = k) = e^{-\lambda} \frac{\lambda^k}{k!}$	$k = 0, 1, 2, \dots$	λ	λ	$\exp\{\lambda(e^t - 1)\}$
Geometric	$0 < p \leq 1$	$P(X = k) = p(1 - p)^{k-1}$	$k = 1, 2, \dots$	$\frac{1}{p}$	$\frac{1 - p}{p^2}$	$\frac{pe^t}{1 - (1 - p)e^t}$
Geometric, zero-based	$0 < p \leq 1$	$P(X = k) = p(1 - p)^k$	$k = 0, 1, 2, \dots$	$\frac{1 - p}{p}$	$\frac{1 - p}{p^2}$	$\frac{p}{1 - (1 - p)e^t}$
Negative Binomial	$r \geq 1, 0 < p \leq 1$	$P(X = k) = \binom{k-1}{r-1} p^r (1 - p)^{k-r}$	$k = r, r + 1, \dots$	$\frac{r}{p}$	$\frac{r(1 - p)}{p^2}$	$\left(\frac{pe^t}{1 - (1 - p)e^t} \right)^r$
Negative Binomial, failures before r successes	$r \geq 1, 0 < p \leq 1$	$P(X = k) = \binom{r+k-1}{r-1} p^r (1 - p)^k$	$k = 0, 1, 2, \dots$	$\frac{r(1 - p)}{p}$	$\frac{r(1 - p)}{p^2}$	$\left(\frac{p}{1 - (1 - p)e^t} \right)^r$
Hypergeometric w, b, n		$P(X = k) = \frac{\binom{w}{k} \binom{b}{n-k}}{\binom{w+b}{n}}$	$\max(0, n - b) \leq k \leq \min(n, w)$	$\frac{nw}{w + b}$	$\frac{w + b - n}{w + b - 1} n \frac{w}{w + b} \left(1 - \frac{w}{w + b} \right)$	$\sum_k e^{tk} \frac{\binom{w}{k} \binom{b}{n-k}}{\binom{w+b}{n}}$
Negative Hypergeometric	w, b, r	$P(X = k) = \frac{\binom{r+k-1}{r-1} \binom{w+b-r-k}{w-r}}{\binom{w+b}{w}}$	$k = 0, 1, \dots, b$	$\frac{rb}{w + 1}$	$\frac{rb(w + b + 1)(w - r + 1)}{(w + 1)^2(w + 2)}$	$\sum_{k=0}^b e^{tk} \frac{\binom{r+k-1}{r-1} \binom{w+b-r-k}{w-r}}{\binom{w+b}{w}}$

Continuous Distributions

Name	Parameters	Probability Density Function	Support	Mean	Variance	MGF
Uniform	$a < b$	$f_X(x) = \frac{1}{b-a}$	$a < x < b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{tb} - e^{ta}}{t(b-a)}$
Normal	$\mu \in \mathbb{R}, \sigma^2 > 0$	$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$	$-\infty < x < \infty$	μ	σ^2	$\exp\left\{\mu t + \frac{\sigma^2 t^2}{2}\right\}$
Log-Normal	$\mu \in \mathbb{R}, \sigma^2 > 0$	$f_X(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left\{-\frac{(\log x - \mu)^2}{2\sigma^2}\right\}$	$x > 0$	$e^{\mu+\sigma^2/2}$	$e^{2\mu+\sigma^2}(e^{\sigma^2} - 1)$	DNE
Exponential	$\lambda > 0$	$f_X(x) = \lambda e^{-\lambda x}$	$x \geq 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$\frac{\lambda}{\lambda - t}$
Gamma	$r > 0, \lambda > 0$	$f_X(x) = \frac{\lambda^r}{\Gamma(r)} x^{r-1} e^{-\lambda x}$	$x \geq 0$	$\frac{r}{\lambda}$	$\frac{r}{\lambda^2}$	$\left(\frac{\lambda}{\lambda - t}\right)^r$
Beta	$a, b > 0$	$f_X(x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1} (1-x)^{b-1}$	$0 < x < 1$	$\frac{a}{a+b}$	$\frac{ab}{(a+b)^2(a+b+1)}$	
Weibull	$\alpha, \lambda > 0$	$f_X(x) = \alpha \lambda x^{\alpha-1} e^{-\lambda x^\alpha}$	$x > 0$	$\frac{\Gamma(1+1/\alpha)}{\lambda^{1/\alpha}}$	$\frac{\Gamma(1+2/\alpha)}{\lambda^{2/\alpha} \mu^2} -$	$-$
Chi-Square	$n > 0$	$f_X(x) = \frac{1}{2^{n/2}\Gamma(n/2)} x^{n/2-1} e^{-x/2}$	$x > 0$	n	$2n$	$(1-2t)^{-n/2}$
Student- t	$n > 0$	$f_X(x) = \frac{\Gamma((n+1)/2)}{\sqrt{n\pi}\Gamma(n/2)} \left(1 + \frac{x^2}{n}\right)^{-(n+1)/2}$	$-\infty < x < \infty$	$0, n > 1$	$\frac{n}{n-2}, n > 2$	DNE

Moment-Generating Function

The moment-generating function is

$$M_X(t) = \mathbb{E}(e^{tX}).$$

When it exists near $t = 0$, it can be used to compute moments:

$$\mathbb{E}(X^n) = M_X^{(n)}(0),$$