Probability Distribution Reference Sheet

MATH 394: Probability I (Summer 2026)

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Discrete Distributions I

Quantity	Bernoulli	Binomial	Poisson	Geometric	Geometric, zero-based
Parameters	$0 \le p \le 1$	$n \ge 1,\ 0 \le p \le 1$	$\lambda > 0$	$0 < p \le 1$	$0 < p \le 1$
PMF	$P(X=k) = p^k(1-p)^{1-k}$	$P(X=k) = \binom{n}{k}p^k(1-p)^{n-k}$	$P(X=k) = e^{-\lambda}\frac{\lambda^k}{k!}$	$P(X=k) = p(1-p)^{k-1}$	$P(X=k) = p(1-p)^k$
Support	$k = 0, 1$	$k = 0, \dots, n$	$k = 0, 1, 2, \dots$	$k = 1, 2, \dots$	$k = 0, 1, 2, \dots$
Mean	p	np	λ	$\frac{1}{p}$	$\frac{1-p}{p}$
Variance	$p(1-p)$	$np(1-p)$	λ	$\frac{1-p}{p^2}$	$\frac{1-p}{p^2}$
MGF	$1 - p + pe^t$	$(1-p+pe^t)^n$	$\exp\{\lambda(e^t-1)\}$	$\frac{pe^t}{1-(1-p)e^t}$	$\frac{p}{1-(1-p)e^t}$

Discrete Distributions II

Quantity	Negative Binomial	Negative Binomial, failures before r successes	Hypergeometric	Negative Hypergeometric
Parameters	$r \ge 1,\ 0 < p \le 1$	$r \ge 1,\ 0 < p \le 1$	w, b, n	w, b, r
PMF	$P(X=k) = \binom{k-1}{r-1}p^r(1-p)^{k-r}$	$P(X=k) = \binom{r+k-1}{r-1}p^r(1-p)^k$	$P(X=k) = \frac{\binom{w}{k}\binom{b}{n-k}}{\binom{w+b}{n}}$	$P(X=k) = \frac{\binom{r+k-1}{r-1}\binom{w+b-r-k}{w-r}}{\binom{w+b}{w}}$
Support	$k = r, r+1, \dots$	$k = 0, 1, 2, \dots$	$\max(0, n-b) \le k \le \min(n, w)$	$k = 0, 1, \dots, b$
Mean	$\frac{r}{p}$	$\frac{r(1-p)}{p}$	$\frac{nw}{w+b}$	$\frac{rb}{w+1}$
Variance	$\frac{r(1-p)}{p^2}$	$\frac{r(1-p)}{p^2}$	$\frac{w+b-n}{w+b-1}\, n\frac{w}{w+b}\left(1-\frac{w}{w+b}\right)$	$\frac{rb(w+b+1)(w-r+1)}{(w+1)^2(w+2)}$
MGF	$\left(\frac{pe^t}{1-(1-p)e^t}\right)^r$	$\left(\frac{p}{1-(1-p)e^t}\right)^r$	$\sum_k e^{tk}\frac{\binom{w}{k}\binom{b}{n-k}}{\binom{w+b}{n}}$	$\sum_{k=0}^{b} e^{tk}\frac{\binom{r+k-1}{r-1}\binom{w+b-r-k}{w-r}}{\binom{w+b}{w}}$

Continuous Distributions I

Quantity	Uniform	Normal	Log-Normal	Exponential	Gamma
Parameters	$a < b$	$\mu \in \mathbb{R},\ \sigma^2 > 0$	$\mu \in \mathbb{R},\ \sigma^2 > 0$	$\lambda > 0$	$r > 0,\ \lambda > 0$
PDF	$f_X(x) = \dfrac{1}{b-a}$	$f_X(x) = \dfrac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\dfrac{(x-\mu)^2}{2\sigma^2}\right\}$	$f_X(x) = \dfrac{1}{x\sigma\sqrt{2\pi}} \exp\left\{-\dfrac{(\log x-\mu)^2}{2\sigma^2}\right\}$	$f_X(x) = \lambda e^{-\lambda x}$	$f_X(x) = \dfrac{\lambda^r}{\Gamma(r)} x^{r-1} e^{-\lambda x}$
Support	$a < x < b$	$-\infty < x < \infty$	$x > 0$	$x \geq 0$	$x \geq 0$
Mean	$\dfrac{a+b}{2}$	μ	$e^{\mu+\sigma^2/2}$	$\dfrac{1}{\lambda}$	$\dfrac{r}{\lambda}$
Variance	$\dfrac{(b-a)^2}{12}$	σ^2	$e^{2\mu+\sigma^2}(e^{\sigma^2}-1)$	$\dfrac{1}{\lambda^2}$	$\dfrac{r}{\lambda^2}$
MGF	$\dfrac{e^{tb}-e^{ta}}{t(b-a)}$	$\exp\left\{\mu t + \dfrac{\sigma^2 t^2}{2}\right\}$	DNE	$\dfrac{\lambda}{\lambda - t}$	$\left(\dfrac{\lambda}{\lambda - t}\right)^r$

Continuous Distributions II

Quantity	Beta	Weibull	Chi-Square	Student-t
Parameters	$a, b > 0$	$\alpha, \lambda > 0$	$n > 0$	$n > 0$
PDF	$f_X(x) = \dfrac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1}(1-x)^{b-1}$	$f_X(x) = \alpha\lambda x^{\alpha-1} e^{-\lambda x^\alpha}$	$f_X(x) = \dfrac{1}{2^{n/2}\Gamma(n/2)} x^{n/2-1} e^{-x/2}$	$f_X(x) = \dfrac{\Gamma((n+1)/2)}{\sqrt{n\pi}\Gamma(n/2)} \left(1 + \dfrac{x^2}{n}\right)^{-(n+1)/2}$
Support	$0 < x < 1$	$x > 0$	$x > 0$	$-\infty < x < \infty$
Mean	$\dfrac{a}{a+b}$	$\dfrac{\Gamma(1+1/\alpha)}{\lambda^{1/\alpha}}$	n	$0,\ n > 1$
Variance	$\dfrac{ab}{(a+b)^2(a+b+1)}$	$\dfrac{\Gamma(1+2/\alpha)}{\lambda^{2/\alpha}} - \mu^2$	$2n$	$\dfrac{n}{n-2},\ n > 2$
MGF	–	–	$(1-2t)^{-n/2}$	DNE

Moment-Generating Function

The moment-generating function is

$$M_X(t) = \mathbb{E}(e^{tX}).$$

When it exists near $t = 0$, it can be used to compute moments:

$$\mathbb{E}(X^n) = M_X^{(n)}(0).$$