Probability Distribution Reference Sheet

MATH 394: Probability I (Summer 2026)

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Discrete Distributions

Name	Parameters	Probability Mass Function	Support	Mean	Variance	MGF
Bernoulli	$0 \le p \le 1$	$P(X=k) = p^k(1-p)^{1-k}$	$x = 0, 1$	p	$p(1-p)$	$1-p+pe^t$
Binomial	$n \ge 1,\ 0 \le p \le 1$	$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$	$k = 0, \ldots, n$	np	$np(1-p)$	$(1-p+pe^t)^n$
Poisson	$\lambda > 0$	$P(X=k) = e^{-\lambda} \frac{\lambda^k}{k!}$	$k = 0, 1, 2, \ldots$	λ	λ	$\exp\{\lambda(e^t-1)\}$
Geometric	$0 < p \le 1$	$P(X=k) = p(1-p)^{k-1}$	$k = 1, 2, \ldots$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{pe^t}{1-(1-p)e^t}$
Geometric, zero-based	$0 < p \le 1$	$P(X=k) = p(1-p)^k$	$k = 0, 1, 2, \ldots$	$\frac{1-p}{p}$	$\frac{1-p}{p^2}$	$\frac{p}{1-(1-p)e^t}$
Negative Binomial	$r \ge 1,\ 0 < p \le 1$	$P(X=k) = \binom{k-1}{r-1} p^r (1-p)^{k-r}$	$k = r, r+1, \ldots$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$	$\left(\frac{pe^t}{1-(1-p)e^t}\right)^r$
Negative Binomial, failures before r successes	$r \ge 1,\ 0 < p \le 1$	$P(X=k) = \binom{r+k-1}{r-1} p^r (1-p)^k$	$k = 0, 1, 2, \ldots$	$\frac{r(1-p)}{p}$	$\frac{r(1-p)}{p^2}$	$\left(\frac{p}{1-(1-p)e^t}\right)^r$
Hypergeometric	w, b, n	$P(X=k) = \frac{\binom{w}{k}\binom{b}{n-k}}{\binom{w+b}{n}}$	$\max(0, n-b) \le k \le \min(n, w)$	$\frac{nw}{w+b}$	$\frac{w+b-n}{w+b-1} n \frac{w}{w+b}\left(1-\frac{w}{w+b}\right)$	$\sum_k e^{tk} \frac{\binom{w}{k}\binom{b}{n-k}}{\binom{w+b}{n}}$
Negative Hypergeometric	w, b, r	$P(X=k) = \frac{\binom{r+k-1}{r-1}\binom{w+b-r-k}{w-r}}{\binom{w+b}{w}}$	$k = 0, 1, \ldots, b$	$\frac{rb}{w+1}$	$\frac{rb(w+b+1)(w-r+1)}{(w+1)^2(w+2)}$	$\sum_{k=0}^{b} e^{tk} \frac{\binom{r+k-1}{r-1}\binom{w+b-r-k}{w-r}}{\binom{w+b}{w}}$

Continuous Distributions

Name	**Parameters**	**Probability Density Function**	**Support**	**Mean**	**Variance**	**MGF**
Uniform	$a < b$	$f_X(x) = \frac{1}{b-a}$	$a < x < b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{tb} - e^{ta}}{t(b-a)}$
Normal	$\mu \in \mathbb{R},\ \sigma^2 > 0$	$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$	$-\infty < x < \infty$	μ	σ^2	$\exp\left\{\mu t + \frac{\sigma^2 t^2}{2}\right\}$
Log-Normal	$\mu \in \mathbb{R},\ \sigma^2 > 0$	$f_X(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left\{-\frac{(\log x - \mu)^2}{2\sigma^2}\right\}$	$x > 0$	$e^{\mu + \sigma^2/2}$	$e^{2\mu+\sigma^2}(e^{\sigma^2} - 1)$	DNE
Exponential	$\lambda > 0$	$f_X(x) = \lambda e^{-\lambda x}$	$x \geq 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$\frac{\lambda}{\lambda - t}$
Gamma	$r > 0,\ \lambda > 0$	$f_X(x) = \frac{\lambda^r}{\Gamma(r)} x^{r-1} e^{-\lambda x}$	$x \geq 0$	$\frac{r}{\lambda}$	$\frac{r}{\lambda^2}$	$\left(\frac{\lambda}{\lambda - t}\right)^r$
Beta	$a, b > 0$	$f_X(x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1}(1-x)^{b-1}$	$0 < x < 1$	$\frac{a}{a+b}$	$\frac{ab}{(a+b)^2(a+b+1)}$	–
Weibull	$\alpha, \lambda > 0$	$f_X(x) = \alpha\lambda x^{\alpha-1} e^{-\lambda x^\alpha}$	$x > 0$	$\frac{\Gamma(1+1/\alpha)}{\lambda^{1/\alpha}}$	$\frac{\Gamma(1+2/\alpha)}{\lambda^{2/\alpha}} - \mu^2$	–
Chi-Square	$n > 0$	$f_X(x) = \frac{1}{2^{n/2}\Gamma(n/2)} x^{n/2-1} e^{-x/2}$	$x > 0$	n	$2n$	$(1-2t)^{-n/2}$
Student-t	$n > 0$	$f_X(x) = \frac{\Gamma((n+1)/2)}{\sqrt{n\pi}\Gamma(n/2)} \left(1 + \frac{x^2}{n}\right)^{-(n+1)/2}$	$-\infty < x < \infty$	$0,\ n > 1$	$\frac{n}{n-2},\ n > 2$	DNE

Moment-Generating Function

The moment-generating function is

$$M_X(t) = \mathbb{E}(e^{tX}).$$

When it exists near $t = 0$, it can be used to compute moments:

$$\mathbb{E}(X^n) = M_X^{(n)}(0),$$